THE TEACHING OF MATHEMATICS TO YOUNG CHILDREN.

That skill in handling numbers is one of the fundamental bases on which to rear an educational structure is no new idea. Number has formed a part of the most scanty and elementary schemes of education through all historic time, and we may therefore assume that its value is undisputed even by those unable to realise it in exact terms of intellectual training and power.

Those, however, who are deeply interested in the teaching of the science of number realise that, even though it may never be of practical use to the student, yet a true knowledge of this subject will give him such important knowledge as will stand him in good stead in his future dealings with men and affairs. To them no apology is necessary for the exceeding care which we consider must be bestowed on teaching children to really "think mathematically."

Taking as our working definition that "education is an atmosphere, a discipline, a life," it follows that we realise that education must surround and be a part of the child from his infancy; but until he is ready for school at the end of his sixth year it is to be an education by means of his senses, of his unstudied games, by means of his natural and not of an artificially prepared environment.

The conscious teaching then of number, as of other definite lines of thought, is to be begun in the schoolroom with a pupil whose age is not less than six years.

School Mathematics.—When children begin their regular school course, lessons last for two hours, or two-and-a-half hours, every morning, with a long interval; *number* for 20 minutes a day is one of the lessons. We generally find that the children, when they enter school, are able to count, but know nothing of the properties of numbers.

The number *one* is taken during the first lesson; the children point out to the teacher *one* window, *one* fireplace, *one* piano; in fact, everything in the room which exists singly; then the symbol for one is learnt. Whenever we see a stroke 1 we know that it stands for one of something. The children pick out the ones from groups of figures, and finally learn to write one; getting it as straight and perfect as possible.

The children have a small blackboard and piece of chalk each, and on these they first write the numbers; afterwards a book ruled in 1/4 inch or 1/2 inch squares and a lead pencil are requisitioned. The next number to one is two, as the child probably knows; he learns then to write '2,' first on his board,
[p 2]
and then in his book; picks out 2 from a group of figures, and does little sums involving the number 2. Three is taken in the same way; and then four, which the pupil must realise is made up of two twos, or of 3 and 1, by very simple little problems such as will readily suggest themselves to any teacher. He learns to count up to 4, and backwards from 4; thus realising slowly the idea of a series of symbols denoting a series of quantities whose magnitudes continue to grow greater. The idea of an *order* of things, which is conveyed by a number, is perhaps grasped most easily by counting a series of things; and that of the relative magnitudes represented by numbers by the little sums in addition and subtraction.

In this way all the numbers from one to nine are learnt, the examples becoming more numerous as the numbers grow larger, and involving, besides simple subtraction, simple factors such as two threes make six, and three threes make nine. Each number is begun from a concrete set of things, beads, &c., and several questions are asked and answered with the help of the beads. Then these are put away, and for the next lesson work is done on the number without the aid of the concrete.

When several of the numbers have been learnt the meanings of the signs +, -, and = are explained to the child; + means "is added to," or "is put together with," – means "is taken away from," and = means "is the same thing as." Now we have the added joy of being able to write sums in our books. This is always considered a privilege, and is only indulged in on mornings when the children are working well, and during the final lesson on some particular number. Writing is still a laborious effort, and is apt to take attention away from the most important matter in hand. The sums are of course always worked orally first, and then written down, *e.g.*, if your little sister is two years old now, how old will she be in two more years? When the answer 4 has been obtained the children write in their books 2 + 2 = 4; then they read it; two years added to two years make four years. This writing of sums, however, is very sparingly used, and all the work is oral.

During this stage too we give occasional examples dealing with pure number; there are mornings when the little ones are bright and eager, and more than ever anxious to do innumerable sums; this is an opportunity to be seized by the teacher; let us leave the boxes of beads and counters alone, let us even leave out sheep and motor cars, and have nothing but numbers. "How much left if you take 3 from 5?" "How much to be added to 4 to make 7?," and so on, quick question and quick answer, all easy and simple, so that the children may feel at home with the numbers, and feel that they have a real grasp of them, for, though the ability to work with pure number is undoubtedly a function of some minds only, yet it, like an ear for music, can, to a certain extent, be cultivated, to a very limited extent it may be, but even that is worth striving after with our pupils.

[p 3]

The Number Ten.—This number we of course learn just as we did the other numbers, *i.e*., working out addition and subtraction sums involving its use and analysis; then we learn the meaning of the word *unit* and here a few match-sticks seem to serve as the best vehicle to convey the idea we wish to impart. "When we have 10 things we tie them together and call it a 'ten bundle.'" Count out several sets of 10 sticks each and tie them together, now we have several "ten bundles," and each of the sticks in a bundle is called a *unit*. Now count out, 1 unit, 2 units, 3 units, 4 units . . . . 9 units, 10 units, and tie them together. This is done several times; and, as one must be careful not to confuse in the child's mind that which is general with that which is particular, let the counting out be done with beads, buttons, pencils, &c., the beads and buttons being threaded in sets of 10 and the pencils, &c., tied together. The name "ten *bundle*" is used for each of these.

When this convention has become habitual to the pupils, it will be time to write the number ten; it is written on the squared paper, one square having 1 written in it for *one* ten bundle, and the next square 0 for "no units"; now all the other numbers from 1 to 9 are written down under one another, and then 10 is written so that 0 comes under 9, and the 1 is out by itself (to the left) in the "ten bundle" square.

After 10 is thoroughly mastered the numbers from 11 to 20 are always learnt very quickly as 1 ten bundle and 1 unit; 1 ten bundle and 2 units, &c., &c., use being made of the bundles of matches, threads of beads, &c., though these must be dispensed with as soon as possible. The children practise counting too, both backwards and forwards, and learn to write the numbers in columns so that the idea of the local value of the digits may be impressed upon their minds.

We try at this stage, and indeed at any stage in the analysis of numbers from 10 to 100, to obtain from the children themselves the composition of each new number that occurs, *e.g*., we know all numbers from 10 to 13; we have had 10 and no units; 10 and 1 unit, called eleven; 10 and 2 units; 10 and 3 units; then the next number will have to be 10 and 4 units, the next 10 and 5 units, and so on. We find that this sort of counting is necessary, as the children are apt to get an idea of a number by itself, and are unable to realise its position with respect to other numbers.

After the number 12 we have to stop for a while, as we are here able to introduce money; for, though it makes a break in the proper sequence of lessons in numbers from 10 to 20, yet it provides such a valuable teaching asset for the future that it is worth the break in the train of thought we have pursued so far.

We are all provided with purses containing shillings, sixpences, and pennies, and after obtaining from the children the information that lots and lots of pennies would be very heavy to carry about and take a long time to count, we show how instead of six pennies we have a silver coin and we call

[p 4]

it sixpence. Suppose we went into a shop and bought six 1*d*. pieces of chocolate; then instead of counting out six pennies we should just give the shopman *one* coin—one of our sixpences; if we bought 12 penny pieces we might count out 12 pennies (they are counted out by the children), or else we might have,—how many sixpences? (Each set of six pennies is replaced by a 6*d*.); we might have two sixpences, or another coin which we call a shilling and which may be used for two sixpences, or 12 pennies.

After this introduction we have practice in changing from pennies to shillings and sixpences, and *vice versa, e.g*. "count " [sic] out 18 pennies, we don't want to carry so much, so we change " [sic] 12 of them into a shilling—count how many are left." "Six." "Yes, so that we may have a sixpence instead of those pennies, " [sic] and we should only take out with us two small coins instead " [sic] of 18 large ones." Or, "count out 13 pennies, re-place 12 " [sic] by one shilling, we have one penny over, so that we have " [sic] two coins instead of 13: count out eight pennies, a sixpence " [sic] and two pennies," and so on. As the children deal with numbers larger than 12 the work has to be purely experimental.

A great many simple sums on money follow the experimental work, the numbers involved never exceeding 12; *e.g*., I bought three penny balls, two-pennyworth of toffee, and two two-penny balls of string. What change did the shopman give me out of a shilling? Or; I won a prize of 5*s*. and gave 2*d*. out of every shilling to my brother. How much did I give him?

We are now able to use money sums to provide us with examples in dealing with all the other numbers; money sums appeal to children, and at this early stage, as well as a little later on, questions involving money appear to be worked very easily.

The number 20 is easily explained as 2 ten bundles, and one finds that the path from 20 to 100 is a smooth one for the child of ordinary intelligence. From 30 onwards the numbers are taken in sets of ten, and frequent practice in counting and in writing the back figures is given. At the number 20 the children are introduced to the sovereign, and at 24 to the two-shilling piece, and at 30 to the half-crown.

Examples in pure number are also given, such as—add 25 and 41—we add the ten bundles first:—

2 ten bundles + 4 ten bundles = 6 ten bundles, and
5 units + 1 unit = 6 units,
Answer = 66,

these are the steps followed in working; in the pupil's book the sum is written as 41 + 25 = 66.

The analysis of numbers from 1 to 100 occupies the first year; with the second year we begin the four rules and tables. The work carried on does not claim to be original, but is simply a modification of existing methods of teaching, and is taken from existing and well-known text-books. I should like, if I may, to mention what appear to us to be some of the principal features of the course of teaching in these books for the
[p 5]
benefit of those who have perhaps not seen them for themselves.

These features are:—

(1.) The very thorough way in which the work is done; nearly the whole of the first year's work consists of the analysis of numbers from one to a thousand, every one of which is approached from all directions, and exhausted of all its possibilities by the pupil and teacher. The synthesis from pairs of smaller numbers, factors, fractional parts, application to sums of money, to weights and measures (at 60 the pupil begins to work with hours and minutes, at 36 with inches and yards), are all taken with each of the numbers to which they apply, and particular terms, such as dozen and score, are introduced. For instance, the number forty is dealt with thus:—

40 = 39 + 1.
= 38 + 2.
= 37 + 3.
= 36 + 4.
= 35 + 5.
= 34 + 6.
= 33 + 7.
= 32 + 8.
= 31 + 9.
= 30 + 10 = 10 + 10 + 10 + 10 = 4 x 10.
40 = 4 x 10 = 10 x 4.
= 5 x 8 = 8 x 5.
= 20 x 2 = 2 x 20 = two score.

10 ≠ [1]40	4 times.
4 ≠ 40	10 times.
5 ≠ 40	8 times.
8 ≠ 40	5 times.
20 ≠ 40	2 times (twice.)
2 ≠ 40	20 times.

The half of 40 is 20. The fifth of 40 is 8.

The quarter of 40 is 10. The eighth of 40 is 5.
The tenth of 40 is 4. The twentieth of 40 is 2 [sic]
Then examples such as:—
(*a*) How many shillings in £2? Cwts. in 2 tons?
(*b*) How many horses have 40 legs between them?
(*c*) If one book costs 8 shillings, how many can I buy for £2, &c., &c.
(2.) The four rules are introduced all together from the very beginning by means of little problems, *e.g*., "Out of " [sic] a journey of ten miles I have walked seven miles. " [sic] How much further have I to go?" or, "How many twopenny balls for a shilling?" or, "How many legs
[p 6]
have four sparrows?" The four signs +, -, x, ≠, are given to the children almost at the beginning of the course, they are told the meanings of the signs and are given exercises on their interpretation. Terms such as subtrahend, multiplicand, &c., are given as soon as the children arrive at the sections particularly devoted to subtraction, multiplication, &c.
(3.) A special apparatus particularly devised is largely used throughout the course.

Even this very scanty indication of the lines of the work will perhaps suffice to show how tempting such a course is to the teacher of young children; it takes the subject in a fascinating and exhaustive way and appears at a first reading to be the ideal course of work laid down for the beginner; but it has from experience been proved to possess certain disadvantages for which reason it is used with these alterations—

(1.) The whole use of the special apparatus is omitted. The use of complicated apparatus specially designed to teach the child certain facts about numbers tends to form in his mind an iron connection between the facts and the apparatus; he is unable to separate them or to realise that the facts are general. We therefore think it advisable, where apparatus is necessary, to use match sticks, buttons, pencils, everyday articles, and a sufficient number of them.
(2.) The examples given, though of an interesting nature, are often much too hard for the children. Their acquaintance with the science of number is still in its first stages; their work is largely oral, and the examples we find it best to give them have therefore to necessitate for their solution one arithmetical operation performed once only; addition, subtraction, or whatever it may be, *e.g*., "John had 1*s*. more than Mary, he gave her 1*d*. How much had he then more than Mary?" is too difficult a question for a pupil who is just unravelling the mysteries of the number 12.
(3.) All tables of weights and measures of time or length seem better omitted until later on. To be told that there are 20 cwt. in a ton cannot convey much to a child of six; and the divisions of time are an abstraction that will convey still less. We keep therefore simply to sums involving money and familiar everyday objects.
(4.) The signs x and ≠ are not given until the pupil arrives at the multiplication table, and terms such as subtrahend, addenda, are omitted.
(5.) The later numbers are not dealt with so exhaustively; about a week (or 10 days at most) would be spent in learning about the numbers 40-49 for instance. The children do no homework at all, and their arithmetic is therefore very largely oral work.

We have found that the children, when taught entirely on this system, become very proficient in the analysis of numbers,
[p 7]
but that they have, as it were, specialised in that branch of the science, and are unable to work in any other way; problems, if presented as the analysis of a given number, are easily solved, but not when presented in other ways. Books, giving such courses of work, however, are a very valuable guide and assistance in our work.

Addition and Subtraction.—Here the first sums given are those dealing with money. Addition of pence, *e.g*., 7*d*. + 4*d*. and then 7*d*. + 6*d*. + 2*d*. is followed by addition of shillings and pence, *e.g*., 4*s*. 3*d*. + 2*s*. 6*d*. and then 4*s*. 9*d*. + 17*s*. 4*d*., then of pounds, shillings, and pence, the numbers of £ being small:—

£ *s*. *d*.
e.g., 9 10 5
3 17 8
______
Answer 13 8 1, 8 pennies and 4 pennies make 1 shilling, and leave 1 penny over; 1 shilling and 17 shillings make 18 shillings, which, with 2 shillings, make 1£., and leave 8 shillings over; £1 + £3 + £9 = £13. Our answer is, therefore, £13 8*s*. 1*d*.;
and, finally, farthings are introduced. After the children have worked some little time at the addition of money, sums on pure number are given them; *e.g*., 674 + 215; this is to be written in a new way:—
674
215
___
___
the hundreds, tens, and units in their own places. 4 units + 5 units = 9 units, to be written in the units' place. 1 ten + 7 tens = 8 tens, to be written in the tens' place. 2 hundreds + 6 hundreds = 8 hundreds, to be written in the hundreds' place.
The finished sum is then :—
674
215
___
889 Answer.
___
The next step is a sum like:—
519
392
___
___

In a case like this the children are taught to add in tens, *e.g.*, 2 units + 8 units = 10 units and 1 = 11 units, *i.e.*, 1 unit

[p 8]
for the units' place and a ten to be added on to the others. 1 ten + 9 tens = 10 tens, or one hundred, and there is still one ten. We therefore have one hundred and one ten; and the one hundred + three hundreds + five hundreds give us nine hundreds:—

519
392
___
911 Answer.
___

After this longer sums may be given, with three, or four, or more sets of figures, the children always adding in tens and thus attaining a mechanical accuracy. The children are always able after a little judicious questioning to tell us what to do with the 11 units and 11 tens in the sum given above; *i.e.*, to write the 11 units as one in the units' place and put the ten in with the other tens; and with amounts like 13 pennies, which is one shilling and one penny; or 27 shillings, which is a pound and seven shillings, if it were a money question. The analogy between the changing of pennies into shillings, shillings into pounds, and the changing of units into tens, and tens into hundreds, makes the latter seem quite easy and natural to the children. Interesting problems to be worked by the addition of money, or of pure numbers come next, *e.g.*, if a bicycle cost £5 0*s*. 0*d*., a tool-case 2*s*. 6*d*., a bell 1*s*. 9*d*., and a lamp 5*s*. 6*d*., what was the total cost?

Subtraction is introduced as addition was, by little money sums practically presented at first, *e.g.*, (*a*) if I have 6*d*. in my purse and give a porter 2*d*. for carrying a parcel, how much have I left; or (*b*) If I have six nuts and I want 9, how many more must I get? Examples of the nature of (*b*) are advisable, and should be the more numerous because the children can add more easily than they can subtract; and, also, such examples give them the idea of subtraction as the complement of addition; and this view of subtraction is now generally accepted as the right one to be given to beginners, and is used in later lessons on this part of the work. After one or two such examples we begin sums involving shillings, and begin to use the term "take away." Take away 1*s*. 2*d*. from 5*s*. 8*d*., what is left?

s. *d*.
5 8
1 2
___
4 6. Two pennies and 6 pennies make 8 pennies, and 1 shilling + 4 shillings = 5 shillings.
___

Next come sums like this: I had 10 shillings and I paid a man 3*s*. 9*d*., what had I left? Just at first this should be done experimentally; the child attempts to pay 3*s*. 9*d*. out of a purse containing 10 shillings and finds the 9*d*. a difficulty for a little while, though it will soon occur to him to change it into pennies. He then works several sums in his book, mentally changing a
[p 9]
shilling into pennies, and presently pounds into shillings. Even an exceptionally dull child appears to grasp subtraction without any difficulty when it is presented in this way.

When we arrive at subtraction sums involving pure number it is a comparatively simple thing to change tens into units after having already become familiar with the changing of pounds into shillings and shillings into pence; *e.g.*, subtract (we have now learnt that this is the same as "take away") 27 from 41:—

41
27
—
—

We cannot take 7 units from 1 unit—what shall we do? Someone in the class will probably suggest "Turn a 10 into units." We now have 11 units, and 7 + 4 units = 11 units. Then we take 2 tens from 3 tens as 1 has been turned into units; and our sum is:—

41
27
—
14 Answer.
—

If this should not be clear we go back to our bundle of match sticks and break one up into its component units. Though it is generally advisable to give a class only one method of subtraction and the "Decomposition" method just mentioned is the one most easily explained, yet, we may just glance at the method of "Equal Additions" which some teachers prefer.

We begin with our bundle of match sticks. To take 28 from 47, we have before us 4 ten bundles and 7 units—take 8 from 7, we cannot, therefore, instead of untying one of the ten bundles we have, we get a whole new bundle and untie it; we now have 4 tens and 17 units and taking away 8 we have 9 units left. Now we are to take 2 tens away, take them and remember to remove the ten bundle which we put in, *i.e.*, take 3 ten bundles away. This last little "Rule," if we may call it so, is arrived at by the pupil after he has worked several examples. The method of equal additions is eventually a more rapid way of working subtraction and is therefore worth attention. Problems follow involving subtraction of money and pure number and embodying all that has been learnt so far.

Multiplication and Division.—Multiplication is at first presented as an extension of addition, *e.g*., "If 4 children had 6*d*. each, how much had they altogether?" would be worked 6*d*. + 6*d*. + 6*d*. + 6*d*. = 24*d*. = 2*s*. Several examples like this are given before we suggest that it may be written down more shortly, thus 6*d*. x 4, where "x 4" means multiplied by 4, *i.e*., each of the quantities mentioned is to be taken 4 times, so that 6*d*. x 4 means 4 sixpences, 2*s*. x 10 would mean 10 2*s*. pieces,

[p 10]

and so on. We work a few simple questions, getting the children to write them on their blackboards with the multiplication sign and using easy numbers for which a knowledge of the multiplication table is not necessary. These elementary examples give to the children an idea of what "times" indicates and we can then begin Tables.

To help the children to see the rationale of the multiplication table, they at first construct each one for themselves with the teacher's assistance, *e.g.*, suppose it were 4 times, the teacher begins "I write down one 4 on the board with a " [sic] small 1 above it, to show how many fours I have. Then I " [sic] write down another four, how many have I?" "Two." "How much have I now, two fours that is?" "Eight." Put eight down underneath the second 4. Now write down another four, we have three fours or 12, similarly four fours or 16, five fours or 20, and so on to the end of the table, 12 fours or 48, until the whole table stands:—

1 2 3 4 5 6 7 8 9 10 11 12
4 4 4 4 4 4 4 4 4 4 4 4
8 12 16 20 24 28 32 36 40 44 48

The children look at this for some time, visualising it as an aid to committing it to memory, and then say it through several times. The teacher then rubs out several figures here and there in the table and lets the children fill in the gaps thus left. Then the whole table is written out again with several gaps to be filled in by the pupils. The whole table is then said through again by each one.

There is no royal road to the multiplication table; it *must* be learnt by heart. This is a fact which faces every teacher of elementary arithmetic, and which each must prepare for in the best way possible. They must be learnt by each child individually and not in a chorus. The tables are learnt both forwards and backwards as it were, *i.e.*:—

6 times 1 = 6.
6 times 2 = 12, &c., and also
One 6 is 6.
Two 6's are 12.
Three 6's are 18, &c., &c., and are said not in consecutive order, but in a variety of ways, *e.g.*:—
Four 6's are 24.
Three 6's are 18.
Six 6's are 36.
Seven 6's are 42.
Ten 6's are 60, &c.,
and then again in another order.

As each table is mastered examples involving its use and that of previous ones are given, always in the nature of problems beginning with money questions as in addition and subtraction, and proceeding to the manipulation of pure

number; *e.g.*:—A boy was collecting shells, he got 23 every day for 72 days, how many had he then? The sum is written:—

72 23
23
—
—

and the multiplication is worked by 20 first and 3 afterwards. The information as to how it is to be done is obtained from the child, he is able to tell you that you can do it by multiplying by 20 and 3 and adding the products, and that multiplying by 20 is easy because it is just multiplying by 2 and adding a cipher. The finished sum is:—

72 x 23
23
—
1,440

216

—

1,656 Answer

——

Great care is bestowed in seeing that the units, tens, and hundreds are kept in their proper columns all through the sum.

To multiply by hundreds and thousands is also found quite simple when worked in the same way.

Division.—We have now two different meanings to convey: the one an idea of continuous subtraction, the other an idea of fractional parts. The first one can impart by exercises in "sharing" worked with real nuts or pennies just at first; share 12 nuts out into heaps of three, how many boys must there be if each is to have three nuts, how many if each were to have two? How many sticks of chocolate could you buy if each cost 2*d*. and you had 6*d*. to spend? This sharing into heaps of 2 or 3, &c., is called division by 2 or 3, &c. Suppose I asked you to divide 15 into threes, what would you do? Divide it into heaps of 3 and we should have 5 heaps because there are 5 threes in 15. If I divide 8 by 2, how many heaps have I? How many if I divide 10 by 5? 18 by 6? &c., &c. The sign ÷ for division is explained, and a few simple money sums worked next, *e.g*., £6 12*s*. 4*d*. ÷ 2 = £3 6*s*. 2*d*. After this they learn how to divide when the numbers involve tens, and to write the division as short division, *e.g*., 2)48

—

24 *e.g*., two tens and a four; this is demonstrated with "ten bundles" if necessary. We have by now discarded our idea of "heaps" of 2, and realise that 48 ÷ 2 means the number of two's there are in 48. 4 tens ÷ 2 = 2 tens, and 8 ÷ 4 [sic]= 4 units; the answer to 48 ÷ 2 is therefore 24. [p 12]

Long Division.—Some children may not be able to do this, but it is worth trying. We begin with money sums, and a small number as divisor, and say that this is a new way of putting down division sums, so that we can *see* the Remainder after dividing into the pounds, shillings, and pence; or hundreds, tens and units, *e.g*., 4)924(2
8

—

1
4 into 9 hundreds gives 2 hundreds and we have 100 over, *i.e*., we shall have two hundreds in our answer. What shall we do with this 100? Analogy with subtraction will perhaps lead the children to suggest that it should be turned into tens, though we are almost certain to have the question "Why can't you divide 100 by 4?" That would be dividing 100 units, and we must find out how many tens there are in the answer before doing the units. We then have ten tens and two others to add in, *i.e*., 12 tens into which 4 will divide 3 tens times, we shall have 3 *tens* in the answer, and into the 4 units left 4 will divide once, so that the sum stands—
4)924(231. Answer.
8

—

1 hundred
10

___
10 tens
+ 2

___
12 tens
12

___
4 units
4

___
The other aspect of division, *e.g*., that suggesting fractions, is now taken. If I had 8 oranges and gave away half of them, how many would that be? How many if I gave away one-quarter? If I had 9 and gave away one-third? This last probably gets no answer at first. What did giving away one-half mean? Dividing into 2 parts and giving away 1. What then do you think giving away one *third* would mean? Dividing into 3 equal parts and giving one away? Yes, so we divide 9 into 3 equal parts and give 1 away. The third, one-half, one-quarter are written 1/3, 1/2, 1/4, and mean 1 of 3 parts, 1 of 2 parts, 1 of 4 parts, 1/5 would mean one of 5 parts. Now we give many problems, all involving the use of simple apparatus, pennies, pencils, nuts; asking the children for simple fractions, 1/5, 1/3, 1/4 of numbers and letting them obtain the answer by dividing into heaps. Then one obtains from them the information that 1/5 of 20 equals 4 and 20 ÷ 5 = 4, so that 1/4 of 20 means 20 ÷ 4
[p 13]
and one gives them a number of simple sums to be worked orally, and introduces them to the notation for farthings in English money sums. The subject of fractions is left here; it is only touched upon at all in order to give the children a complete notion of division; the subject is not taken up again until the children have mastered decimals.

Weights and Measures.—This part of the subject is reached at the beginning of the average child's ninth year, and is the concluding part of the course in elementary arithmetic. The child weighs and measures for himself and makes his own tables; we let him, dressed in a pinafore, measure out pints and quarts of water, very careful not to spill anything, and weigh pounds and ounces of sand or anything clean and easily handled. Our own weights and measures, of which they have probably heard, are taken first. There are, of course, limitations to schoolroom weighing and measuring: tons and cwt., they are told, are the big weights used when men want to weigh very heavy things like beams of iron; and these weights are added to the table already made from drams to pounds.

After the tables have been made the children read through them once or twice and then have a number of rapid oral questions. How many ounces in two pounds? In 1 lb. 10 ozs.? How many cwts. in 1 ton 6 cwt.? &c., &c., and then a large number of problems to be worked in their books. The metric tables are given as those used in other countries, and the children take to them as ducks to water; every British teacher of arithmetic must bemoan the fact of British weights and measures. With our adoption of a decimal system of coinage and measurement a great deal of what seems such unnecessary labour on his or her part will be saved; the rational and logical aspect of this working in tens seems to appeal to the children as well as to the teacher, and they are always delighted when metric measures are involved in their sums.

The children work practical problems as much as possible at this stage, measuring the girth of trees, the furniture of the schoolroom, &c., and making thus a table of comparative weights and measures; metric expressed in British units and *vice versa*.

Measures of Area.—How should we be able to tell how big the wall is, or the floor? We can tell how long the picture rail is, how *long* and how *wide* the door, but we must get a new way of telling how big it is altogether. Look at a page of your arithmetic book; it is divided up into squares, and you can tell how big it is by counting the squares. Draw a picture of a book 6 squares long and 5 squares wide. How big is it? 30 squares big, because there are 5 rows each containing 6 squares. Draw the top of a pencil box 2 squares wide and 10 squares long; there are 2 rows of 10 squares each; it is 20 squares big. Now suppose we have each square 1 inch long and 1 inch wide, it is then called a square inch: if we had it

1 foot long and 1 foot wide we should call it a square foot, and we measure walls and doors by dividing them up into square feet or square inches. This kind of bigness is called *area*. We draw many rectangular figures on paper divided into square inches, and find the area of each by counting the number of rows and number of squares in each, *e.g*., a rectangle 5 inches long and 2 inches wide.

area = 5 rows of 2 squares each.

= 5 x 2 squares.

= 10 squares or 10 square inches.

Thus the children finally arrive at the rule for finding areas, and then make themselves a table of square yards, feet, and inches. They now measure and find the area of hearthstones, windows, steps, blackboards, &c., and work numerous problems in their books. Metric square measure is taken in the same way, *i.e*., by measuring rectangles divided into centimetre squares, and by measuring the furniture of their schoolroom in metres and centimetres.

Cubic Measure has to be taken in much the same way as square measure, dice or cubes of some sort serving for the preliminary stage, which is shorter than with square measure.

This concludes the fourth year of school life with the ordinary child, and though he has not perhaps progressed very rapidly or “done” many rules and varieties of sums, he has worked thoroughly and discovered much for himself.

During the last year he has begun Geometry, taking the subject quite experimentally and practically, learning how to handle the instruments, to use his eye as a judge of lengths and areas, to know the names and some properties of certain geometrical forms, *e.g*., he discovers by drawing and measurement facts about the intersection of the diagonals of a square and a rectangle, and of parallelograms in general, and the meanings of some geometrical terms, *e.g.*, bisect, perpendiculars, &c., so that when the study of the subject is properly begun he starts with a certain equipment, the foundational ideas are mastered to some extent, and he is able to concentrate his attention on the reasoning out of propositions.

During his sixth and seventh years spent in Class I., he has learnt how to work in carton; to model cubes and chairs and many other things; each model involving the drawing of straight lines of given lengths, the keeping of square corners, and very neat and accurate cutting out. All this serves him in good stead now, his fingers can hold a ruler steady and straight, he draws neat and tidy lines, and is accurate.

To use Ruskin's words, anent Geometry: "You have now " [sic] learned, young ladies and gentlemen, to read, to speak, to think, " [sic] to sing, and to see . . . . . Here is your carpenter's square " [sic] for you, and you may safely and wisely contemplate the " [sic] ground a little, and the measures and laws relating to that,

" [sic] seeing you have got to abide upon it:—and have properly " [sic] looked at the stars; not before then, lest, had you studied the " [sic] ground first, you might perchance never have raised your heads " [sic] from it. Geometry is here considered as the arbitress of all " [sic] laws of practical labour, issuing in beauty."[2]

The Geometry lessons begin with points, and with lines, straight and curved, the children's own definitions being taken if possible; problems are given them which will elicit certain facts about lines, *e.g.*, "draw as many straight lines as you can through a certain point," or "take 3 points and join each one to every other, how many lines have you?" We also give a lot of practice in judging of the lengths of lines both in centimetres and inches, the amount of error always being found; it is all done of course by work on the concrete, the schoolroom and things out of doors provide an endless supply of straight lines. We learn the meaning of "bisect" and "trisect," and try to perform these operations without measurement. The next step is to draw plans to scale. I want to draw a picture of the window on a piece of paper not nearly large enough. How shall I do it? Why, draw it smaller, of course. How much smaller? This invites suggestions of which the best should be taken. Eight times as small? Very well, but how shall I be sure that it is eight times as small? Measure it. How? Measure each side and make each eight times as small. This is done and the window drawn and divided into its component panes, the scale of course is very carefully put in. After this doors, books, &c., are drawn to scale, a plan of the schoolroom made with windows, doors, and fireplace in. Then we get out our atlases and see that all maps are really plans drawn to scale, and we can find out by measuring and using the scale how far places are from each other, though here we must go carefully, as the scale given on a map is not by any means accurate. The scale on all maps must necessarily differ in different directions, and the results obtained can therefore be only approximate, though sufficiently so to be of use and interest to the children. After practice in map-measuring, we have a few problems, such as:—"If I look " [sic] west from my house, I see a lighthouse 4 miles away, if I look ' [sic] west I see a church spire 3 miles away. Draw a plan (scale " [sic] 1 mile to 1 inch), and find out how far the lighthouse is from " [sic] the tower."

The circle comes next, the children practise drawing them, and learn the meaning of radius, centre, circumference, arc, chord, diameter; then they draw circles, concentric, intersecting, &c., &c., and have problems connecting circles and their former work, *e.g.*, a lighthouse whose light has a radius of 4 1/2 miles, is 32 miles from another light, with a radius of 3 miles. Draw a plan to find what space of darkness a ship would have to cross in going from one to the other.

Angles are taken next, and are generally defined as "corners" by the children, they see the 2 ways in which a "corner" may be formed; by 2 lines meeting or by 1 line revolving about another; they also learn that blunt and sharp angles are called obtuse and acute, and what a right angle is like. They also find out how the magnitude of an angle can be tested, and that it is not affected by the length of its arms, for two equal angles can be formed by the opening out of a large and a small pair of compasses, for instance. The use of the protractor comes next, and when the children are proficient in this, they can find out the properties of vertically opposite angles, angles in a circle, &c., by drawing and measuring for themselves. The clock-face provides useful practice in the magnitudes of angles and in drawing circles. Plans are given involving the measuring of angles and distances. The questions must all have some purpose in them; a question like "Draw " [sic] XY 4 inches long, at X draw a line making an angle 58° with " [sic] XY, and at Y a line making an angle an angle [sic] 48° with XY," leaves an unfinished idea in the child's mind and makes his geometry lesson a play at purposeless lines and angles; or makes his work machine-like; he stops like a machine at the drawing of that third line because he sees no reason for stopping there; and he is ready like a machine for the next such pointless demand.

After angles, we are able to take the 16 points of the compass, with many exercises from maps or from plans which the children draw. Work on parallels follows very easily from this; as parallel lines are simply those in the same direction, *e.g*., railway lines, or roads in the neighbourhood; it can be ocularly demonstrated that two people walking always in the same direction and apart, never draw any nearer each other. The names of the angles "alternate," "exterior," and "interior," are learnt: this is really advisable as an aid to the future when the proposition dealing with parallel lines is taken. If the pupils are not already familiar with the names of the angles formed by a pair of parallel lines with a transversal, we find that the proposition involving the properties of these angles invariably proves a source of confusion and difficulty. The children are next introduced to set squares, and shown how to use them, and are taught how to draw parallel lines in this way. Throughout this early part of the work we try to avoid as far as possible any approach to a proposition in the style of Euclid; all the work is a preparation for the logical proving of propositions that is to come; and any approach to it now is apt to result in some of the parrot-work so much affected by former students of Euclid. All we teach them, therefore, at this stage is to bisect a straight line (this being a very simple proposition can be taken as one of the ways in which circles may be used), and perhaps to bisect an angle and make one angle equal to another, though, as these latter mean necessarily
[p 17]
a mechanical effort of memory, it is advisable to omit them with a slow class. Throughout the work *accuracy* in all measurements and drawings is insisted upon, as this is absolutely necessary when so much is done visually.

All the work in Mathematics is greatly assisted by the "geography" walks taken by the children, *i.e*., walks during which exercises in pacing, in compass reading, in judging of heights and distances by eye are given; *e.g*., "make a plan of " [sic] the road from our gate to the market square putting in " [sic] telegraph poles, houses on the route, &c,, [sic] &c.;" this is done by pacing the distances and taking directions with a pocket compass; or, "find the amount of water per minute brought " [sic] down by the stream above and below the water-works; hence " [sic] find the amount of water taken by the works"; this would come in with the arithmetic lessons on cubic capacity.

The rate at which the stream flows is determined by timing a floating stick down a measured length of the bank; and the width and depth are measured at what seem to be the best points for giving an approximately average estimate.

After a year of geometry the children have obtained a good idea of direction and distance, some familiarity with ordinary mathematical instruments, habits of neatness and accuracy which are of value in one's dealings with later Mathematics, and some small power of logical reasoning acquired by the tackling of the problems presented.

One's aim throughout in the teaching of Mathematics in these years is twofold:—

(1.) To prepare the soil from which in after years it may be possible for a good mathematician to spring, *i.e.*, to teach a child the significance of 2 + 2, keeping before one's vision the day when he might be seeking the significance of dy/dx, and

(2.) To make sure that, even if the child's mathematical training should end with his ninth year, yet the intellectual development it has occasioned and the powers it has called forth are such as will be a valuable asset to him in all his future work; for, to quote again from Ruskin, we reveal to him "the Science of Number. " [sic] Infinite in solemnity of use . . . . including, of course, ". . . . the higher abstract mathematics and mysteries of " [sic] numbers, but reverenced especially in its vital necessity " [sic] to the prosperity of families and kingdoms."[3] And, again, "And how often in greater affairs of life the " [sic] arithmetical part of the business must become the " [sic] dominant one! How many and how much have we? " [sic] How many and how much do we want? How constantly does noble Arithmetic of the finite lose itself in

[p 18]

" [sic] base Avarice of the Infinite, and in blind imagination " [sic] of it? In counting of minutes, is our arithmetic ever "solicitous enough? In counting our days, is she ever " [sic] severe enough"?[4]

Here, then, we obtain authority for our hope that our training is to help in the training of a good and capable citizen of his country. Stress is laid then upon one or two points in the general teaching.—

(1.) The children are taught to formulate rules for themselves by working out several examples from first principles, and when the rule *is* formulated to use it immediately to shorten their work, *e.g*., a child works several sums such as, "Find the cost of 12 things at 3*d*. each, 4*d*. each, &c.," and hence formulates for himself the rule that "the number of shillings per dozen is the same as the number of pence apiece," this leads to the habit of investigation so essential to the higher mathematician.

(2.) We insist also upon concentration of thought throughout the lessons which range in duration from 20 minutes at first to 25 minutes in the last year; during that time attention and concentrated thinking are required; the children generally have an easy lesson, such as handicrafts or writing, to follow so that their brains are rested after the effort expended.

To every teacher of this subject it is now clear that the historical presentation of the subject is the easiest and most natural, *i.e*., that it is to be presented to the child as it presented itself to the race; beginning with the concrete and working back to the abstract generalisation; and having as far as possible a practical bearing on matters of everyday life. Thus much is clear, but what is not always kept so distinctly before the teacher is that the concrete in our case is simply the means and not the end: the function of the concrete is to be simply a preparation for the abstract, or a means of symbolic illustration; it must therefore be discontinued as soon as

possible. We find that the children get worried and bored by counters and beads, and work drags wearily on, always the same counters and beads, until the children's attention and interest have both vanished, and the lesson is actively productive of harm. A variety of objects is necessary in these lessons with the concrete, a child may, for instance, learn all about the number 10 from certain cubes of wood, but when the cubes being absent or something else in their place, he is asked about the number 10, all knowledge of it has vanished; 10 pertains specially to cubes and does not exist without them. Once the concrete has been passed, it is better not to go back to it for assistance; it thus becomes just a stage on the way to something fuller, and not a prop to be constantly leaned upon.
[p 19]

We often hear that sums dealing with interesting things like oranges or tops, or dolls, should not be given to young children, as they are apt to fix their attention on the tops or dolls and not on the numbers; this we are inclined to think is entirely the teacher's fault; though the question is always approached by means of a problem, a beginner can soon be taught that for the working out the *numbers* are the primary things; this is made easier by writing in arithmetic books only the figures involved, until the answer is obtained, *e.g*., "I went " [sic] out with 6*d*. and spent 4*d*. on biscuits; I then met a friend, " [sic] who gave me another 6*d*., how much did I come home with?"

The sum would be stated—

6	2
4	6
—	—
2	8 pennies. Answer.
—	—

The answer must be noted as pennies and hailed as an important and interesting conclusion by some remark such as "So I came home richer than I went out."

There are of course several excellent text-books providing intelligent examples and lucid explanations; but at these very early stages of the subject a text-book can be almost entirely dispensed with; rather than consult text-books the teacher must use her ingenuity, must utilize other lessons and walks, &c., to point her definitions and amplify her instruction.

IRENE STEPHENS.

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[1] ≠ = is contained in. This symbol was proposed by Messrs. Sonnenschein and Nesbit. It has not been generally accepted, and unfortunately it resembles closely the sign universally employed for "does not equal."
[2] John Ruskin. Mornings in Florence, being simple studies of Christian Art for English Travellers, p. 134. (London: George Allen, 1894.)
[3] *loc. cit*., pp. 135-137.
[4] *loc. cit*., pp. 135-137.