MATHEMATICS.

By Irene Stephens.

Those of us who work at Elementary Mathematics owe it both to ourselves and to our work to stop sometimes and look at the subject of Mathematics as a whole. We are so apt to get absorbed in our own little patches of drudgery and to forget that they are only patches after all; only the odd bits of mosaic that go to make up a wondrous whole.

Can one not imagine one of Queen Matilda's ladies tiring of her own piece of the Bayeux Tapestry? "I am so weary of sewing in these hundreds of little green stitches," we hear her say, "they are not so very beautiful after all"; and then there comes to this maiden someone with the beautiful picture of the finished tapestry—and the green patch is a patch no longer, it has its own place and its own value; and she, realizing this, thinks no more of green stitches but only of the landscape, part of which it is her high privilege to work upon the canvas.

So we perhaps may hear the children say, or some of us may even say ourselves, "but I am so tired of doing Proportion sums; I can't get them right and what is the use of them anyhow?" and here comes in our need to look at the place of Proportion in the general scheme of things; to reflect upon what it really means. Is not half the beauty in the world a beauty of proportion, beauty of form, that beauty which makes such a soul-stirring appeal to even the most uneducated person who gazes upon one of our great churches or cathedrals? Even in our everyday speech has not "out of proportion" come to mean something which is ugly and unfitting? And it is of the laws which govern this quality that we are learning! Laws, to know whose workings some of the early Greeks would almost have given their lives! The wonderful beauty of Greek buildings and Greek monuments is due we are told to their wonderful *proportions*; the perfect harmony of length and width and height; for the early Greeks strove after the laws of proportion until they had attained to them; and that they were worth attaining to is surely evidenced by the wonderful heritage of beauty the Greek nation has left to us.

[p 139]

This instance of Proportion is taken as just the first which came to one's mind, it occupies no unduly exalted place. Any other section of the subject of Mathematics, if we look at it, not as an entity but simply in its place as part of a whole, must in the same way make us feel that we simply dare not despise, dare not cavil any more at, something at once so dignified and so beautiful.

We are so blasé about numbers and magnitudes in these days, we have had them for such centuries that we take them for granted and never stop to wonder at them. It is good for us to go back to the days of the early philosophers; to put ourselves in the place of one of them, and to look at the Science of Number as he looked at it; it strikes us afresh then with all its mystery and charm. Let us go back to the 6th century, B.C., to put ourselves in the place of the Greek philosopher Pythagoras, "of all men the most assiduous enquirer." We know how he is painted in the fresco of the Spanish Chapel at Florence, where he is depicted under the figure of Arithmetic; for this most assiduous enquirer enquired chiefly into the properties of numbers. The course of his investigations led him to connect his numbers with geometrical figures of various kinds; for one thing he knew that if you were to draw a triangle whose sides were

respectively three, four, and five inches, or feet, or yards, long; then that triangle would always have a right angle between the two shorter sides. One day it dawned on Pythagoras that 3[sic] + $4^2 = 5^2$, or that, 9 + 16 = 25; and then a brilliant inspiration came “since the numbers themselves give me this equality, suppose that I should draw squares on the sides of my triangle. Will the areas of the two smaller squares added together make the area of the larger?” We may imagine the eagerness with which he would set to work to try; and the joy with which he would discover that his inspiration was a true one. For what a wonderful new thing had he found! That space was ruled by number! That squares, divisions of space, had to obey the laws of number! Up to that time Geometry and Arithmetic were two separate sciences, having no connection with one another; and now Pythagoras had found that there was a connection; that with his science of Arithmetic he was lord of space! And not only did numbers govern space; but they governed music too he found; can we wonder then that he should found a school of
[p 140]
philosophy which held as its fundamental creed that “All things are Numbers?” It held that numbers were the final essence to which all sensible things might be reduced, and the way of knowledge was the way of numbers. It was some ripple of this feeling no doubt which caused Plato to put over the door of his Academy the well-known “No entrance to the ungeometrical”; the ungeometrical were not the true searchers after truth, and so he would have none of them.

It need not surprise us that in those early times Philosophy was bound up with Mathematics; for what other field of knowledge could the wise men explore? Mathematics is one of the things that has been waiting for us since the world began; we cannot ever invent anything in Mathematics; we can only discover what is there waiting for the human race to arrive at it, and for those men there was nothing else waiting. All our later knowledge, our theories about sound and electricity, our microscopes and telescopes which bring us to knowledge of the natural world, our ability to rush about in swift ships and motors, each and all of these things rests finally upon a foundation of mathematical theory; and someone had to prepare that foundation or none of these would have been possible.

There is something very beautiful about the unity of the whole Creation thus revealed to us by Mathematics; it is as though a number of wonderful pathways were to converge to a common centre; so that one might begin at the centre and explore more and more of the wonders of the pathways; or one might begin upon a pathway and arrive at the centre to discover that the other ways too have led to it. Let us take for instance the curve known to us as a Conic Section, and discovered by a Greek professor of Mathematics named Menæchmus among whose students was Alexander the Great; we read how impatient of the study of Geometry Alexander the Great was. Here was a burning youth, eager to be up and doing, he to be set down to the study of useless geometrical figures, of what use were they in conquering the world? And yet upon the Conic Section, one of those figures so despised by the great general, the whole of our modern science of Gunnery is founded! Truly, Geometry has been of some use in conquering the world! Menæchmus found that by cutting a cone he got a series of symmetrical figures; the figures that
[p 141]
we get when we throw a ball into the air, or stir our tea; the figures that the great comets, the planets, all those far off celestial bodies are moving in. And we have got them simply by cutting across an ordinary little cone that is so easy to obtain! Think of what is shut up in that

seemingly commonplace solid! The major part of the great science of Geometry for one thing; for the Conic Sections include a point, a pair of straight lines and a circle, besides the parabola, ellipse and hyperbola, and from investigation of the first three comes all the Geometry of points, straight lines, and circles. But, more than this, the cone holds for us the secret of all motion as we have it; for all motion under gravity be it as complicated as possible may be resolved into motion in one or other of the Conic Sections; and the mathematician when he has penetrated their mysteries holds in the hollow of his hand all moving things in the Universe. Knowledge of these curves not only gives us knowledge of the movements of the planets and comets and meteors, and thus some knowledge of the constitution of the Universe; but it enables engineers to build their great bridges, and military bodies to make their perfect preparations for war; in fact, at the centre of that complicated maze called "Modern Civilization" lies one geometrical curve, for all the curves mentioned before are but special cases of the one called the General Conic, and represented by the equation:—

$$ax^2+by^2+2gx+2fy+2hxy+c=0;$$

one which looks simple enough seen thus but is full of significance to those who realize its meaning.

Is there not something utterly superhuman and stupendous about truth of this kind? It is inspiring to think that such fundamentals are actually put within our grasp, given us in such simple form, it fills us with such a sense of power over matter; for here to our hand is the key which will unlock for us all the doors of the things that *are*; not the things we make like literature and history, but those that wait for us patiently through the centuries; and we come to them because we possess the golden key of Numbers—wonderful numbers which we ill-treat in such a shabby way sometimes.

As it is to teachers this is addressed, we must look at our subject from its practical side too. We hear so much nowadays of the historical presentation of mathematics, "give your children

[p 142]

the concrete," we are told, "give them questions which will be *useful*, because that is how the race has learned mathematics; it is a science of utility, and as a science of utility you must present it to your pupils." It all sounds so beautiful and so reasonable, but it can be so misleading. True, it is probable that primitive man learnt to count because he was afraid his neighbour would steal his sheep; true, the science of Geometry had its beginnings in the need for some settlement of the disputes between irate Egyptians whose boundary marks had been washed away by the Nile, and there were diverse opinions upon their proper situation, but mathematics made no progress while it stopped at its utility point; if it had stopped there we should not even have known to-day what a straight line was. The Egyptians had a crude knowledge of right-angled triangles; that is they knew that three cords of lengths 3, 4, and 5, gave you a figure of a particular shape, but beyond this they did not get. It required their Greek visitor Thales with his passionate Greek thirst for "knowledge for the sake of knowledge," with his pure love of knowing, to raise their Geometry to a science, to translate their cords into geometrical lines, and to discover what their ever-recurring figure really was. He soon absorbed all they knew, taught them what he had derived from his purely abstract work on their foundations, and took all this new learning back to Greece, where it was seized [sic] upon by his fellow-countrymen, and added to as we have seen by philosopher after philosopher. That is the true history of Mathematics, Menæchmus never thought his Conic Section was going to be useful, Newton as we know proved his theory of Gravitation and lost the proofs, being perfectly content that he had attained to this discovery and recking little of its utility to the rest of the world. To look therefore at the subject merely as a science of Utility would stultify rather than develop a mathematical sense; use the concrete by all means; but use it very very carefully, only at the very beginning, and only as a support which is to be dispensed with as soon as possible. Love of number for the sake of number is what will help our pupils on, it is this we must seek to cultivate in them; to work sums for the æsthetic pleasure they derive from them; not because they will help them to buy tables and chairs when they are grown up.